

Tensor Gauge Boson Production
in
High Energy Collisions

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Abstract

We calculated the leading-order cross section for the helicity two tensor gauge bosons production in fermion pair annihilation process. We compare this cross section with a similar annihilation processes in QED with two photons in the final state and with two gluons in QCD.

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1 Introduction

Our intention in this article is to calculate leading-order differential cross section for the tensor gauge bosons production in the fermion pair annihilation process. The process is illustrated in Fig.1. and receives contribution from three Feynman diagrams shown in Fig.3. This diagrams are similar to the QED and QCD diagrams for the annihilation processes with two photons or two gluons in the final state. The difference between these processes is in the actual expressions for the corresponding interaction vertices. The corresponding vertices for the tensor bosons can be found through the extension of the gauge principle [9]. The extended gauge principle allows to define a gauge invariant Lagrangian \mathcal{L} for high-rank tensor gauge fields and their cubic and quartic interaction vertices [9, 10, 11]:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_2 + \mathcal{L}'_{2\dots}$$

The Feynman rules for this Lagrangian can be derived from the functional integral over the fermion fields $\psi_i, \psi_i^\mu, \dots$ and over the gauge boson fields $A_\mu^a, A_{\mu\nu}^a, \dots$

Not much is known about physical properties of such gauge field theories [1, 2, 3, 5, 6, 7, 8] and in the present article we shall ignore subtle aspects (ghosts) of functional integral quantization procedure because we limited ourselves to calculating only leading-order tree diagrams. Expanding the functional integral in perturbation theory, starting with the free Lagrangian, at $g = 0$, one can see that free theory contains tensor gauge bosons and fermions of different spins with cubic and quartic interaction vertices [9, 10, 11]. Explicit form of these vertices is presented in [11].

In the next section we shall present the Feynman diagrams for the given process, the expressions for the corresponding vertices, the transition matrix element and unpolarized cross section in the center-of-mass frame. In the third section we shall check that the transition amplitude is gauge invariant, that is, if we take the physical - transverse polarization - wave function for one of the tensor gauge bosons and unphysical - longitudinal polarization - for the second one, the transition amplitude vanishes. This Ward identity expresses the fact that the unphysical - longitudinal polarization states are not produced in the scattering process. In the fourth and fifth sections the squared matrix element is calculated together with traces over Dirac and isotopic matrices for unpolarized particles. In the sixth section we present the final expression for the cross section (6.25) and its comparison with the corresponding cross sections for photons and gluons in QED and QCD.

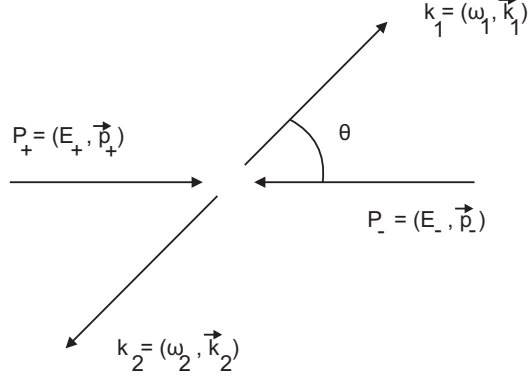


Figure 1: The annihilation reaction $f\bar{f} \rightarrow TT$, shown in the center-of-mass frame. The p_{\pm} are momenta of the fermions $f\bar{f}$ and $k_{1,2}$ are momenta of the tensor gauge bosons TT .

2 Tensor Gauge Bosons Production Amplitude

As we already mentioned in the introduction the process is illustrated in Fig.1. Working in the center-of-mass frame, we make the following assignments:

$$p_- = (E_-, \vec{p}_-), \quad p_+ = (E_+, \vec{p}_+), \quad k_1 = (\omega_1, \vec{k}_1), \quad k_2 = (\omega_2, \vec{k}_2), \quad (2.1)$$

where p_{\pm} are momenta of the fermions f^{\pm} and $k_{1,2}$ momenta of the tensor gauge bosons TT . All particles are massless $p_-^2 = p_+^2 = k_1^2 = k_2^2 = 0$. In the center-of-mass frame the momenta satisfy the relations $\vec{p}_+ = -\vec{p}_-$, $\vec{k}_2 = -\vec{k}_1$ and $E_- = E_+ = \omega_1 = \omega_2 = E$. The invariant variables of the process are:

$$\begin{aligned} s &= (p_+ + p_-)^2 = (k_1 + k_2)^2 = 2(p_+ \cdot p_-) = 2(k_1 \cdot k_2) \\ t &= (p_- - k_1)^2 = (p_+ - k_2)^2 = -\frac{s}{2}(1 - \cos \theta) \\ u &= (p_- - k_2)^2 = (p_+ - k_1)^2 = -\frac{s}{2}(1 + \cos \theta) \end{aligned}$$

where $s = (2E)^2$ and θ is the scattering angle, so that the scalar products can be found in the form

$$\begin{aligned} (p_+ \cdot p_-) &= (k_1 \cdot k_2) = \frac{s}{2} \\ (p_- \cdot k_1) &= (p_+ \cdot k_2) = \frac{s}{4}(1 - \cos \theta) \\ (p_- \cdot k_2) &= (p_+ \cdot k_1) = \frac{s}{4}(1 + \cos \theta). \end{aligned} \quad (2.2)$$

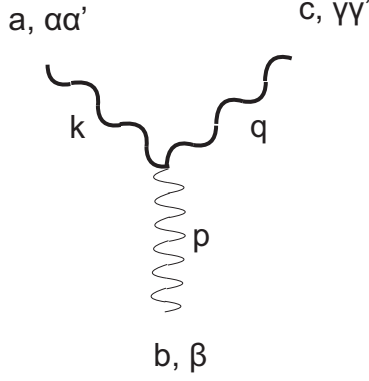


Figure 2: The interaction vertex for vector gauge boson V and two tensor gauge bosons T - the VTT vertex - in non-Abelian tensor gauge field theory [11]. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines. The Lorentz indices $\alpha\acute{}$ and momentum k belong to the first tensor gauge boson, the $\gamma\acute{}$ and momentum q belong to the second tensor gauge boson, and Lorentz index β and momentum p belong to the vector gauge boson.

The Feynman rules for this Lagrangian can be derived from the functional integral over the fermion fields $\psi_i, \bar{\psi}_j, \psi_i^\mu, \bar{\psi}_j^\mu, \dots$ and over the gauge boson fields $A_\mu^a, A_{\mu\nu}^a, \dots$. Here Dirac indices are not shown and i, j and a are indices of the symmetry group G . In this article we shall ignore subtle aspects of functional integral quantization procedure simply because we limited ourselves to calculating only tree diagrams. Expanding the functional integral in perturbation theory, starting with the free Lagrangian, at $g = 0$, one can see that free theory contains a number of free fermions of different spins, each of them have equal dimension $d(r)$ of the representation r : $i, j = 1, \dots, d(r)$ and that the number of free vector- V and tensor- T gauge bosons is equal to the number $d(G)$ of generators of the group G : $a = 1, \dots, d(G)$ [9, 10, 11].

In momentum space the interaction vertex of vector gauge boson V with two tensor gauge bosons T - the VTT vertex - has the form[†] [10, 11]

$$V_{\alpha\acute{} \beta \gamma \acute{}}^{abc}(k, p, q) = -g f^{abc} F_{\alpha\acute{} \beta \gamma \acute{}}, \quad (2.3)$$

where

$$F_{\alpha\acute{} \beta \gamma \acute{}}(k, p, q) = [\eta_{\alpha\beta}(p - k)_\gamma + \eta_{\alpha\gamma}(k - q)_\beta + \eta_{\beta\gamma}(q - p)_\alpha] \eta_{\acute{} \acute{}} - \frac{1}{2} \{ + (p - k)_\gamma (\eta_{\alpha\acute{}} \eta_{\acute{} \beta} + \eta_{\alpha\acute{}} \eta_{\beta \acute{}}) \}$$

[†]See formulas (62), (65) and (66) in [11].

$$\begin{aligned}
& + (k - q)_\beta (\eta_{\alpha\dot{\gamma}} \eta_{\dot{\alpha}\gamma} + \eta_{\alpha\dot{\alpha}} \eta_{\gamma\dot{\gamma}}) \\
& + (q - p)_\alpha (\eta_{\dot{\alpha}\gamma} \eta_{\beta\dot{\gamma}} + \eta_{\dot{\alpha}\beta} \eta_{\gamma\dot{\gamma}}) \\
& + (p - k)_{\dot{\alpha}} \eta_{\alpha\beta} \eta_{\gamma\dot{\gamma}} + (p - k)_{\dot{\gamma}} \eta_{\alpha\beta} \eta_{\dot{\alpha}\gamma} \\
& + (k - q)_{\dot{\alpha}} \eta_{\alpha\gamma} \eta_{\beta\dot{\gamma}} + (k - q)_{\dot{\gamma}} \eta_{\alpha\gamma} \eta_{\dot{\alpha}\beta} \\
& + (q - p)_{\dot{\alpha}} \eta_{\beta\gamma} \eta_{\alpha\dot{\gamma}} + (q - p)_{\dot{\gamma}} \eta_{\alpha\dot{\alpha}} \eta_{\beta\gamma} \}.
\end{aligned} \tag{2.4}$$

The Lorentz indices $\alpha\dot{\alpha}$ and momentum k belong to the first tensor gauge boson, the $\gamma\dot{\gamma}$ and momentum q belong to the second tensor gauge boson, and Lorentz index β and momentum p belong to the vector gauge boson. The vertex is shown in Fig.2. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines.

It is convenient to write the differential cross section for our process, with tensor boson produced into a solid angle $d\Omega$, as

$$d\sigma = \frac{1}{4(p_+ \cdot p_-)} |M|^2 d\Phi, \tag{2.5}$$

where the final-state density for two massless tensor gauge bosons is

$$d\Phi = \int \frac{d^3 k_1}{(2\pi)^3 2\omega_1} \frac{d^3 k_2}{(2\pi)^3 2\omega_2} (2\pi)^4 \delta(p_+ + p_- - k_1 - k_2) = \frac{1}{32\pi^2} d\Omega,$$

so that

$$d\sigma = \frac{1}{2s} |M|^2 \frac{1}{32\pi^2} d\Omega. \tag{2.6}$$

We shall calculate the unpolarized cross section for this reaction, to lowest order in $\alpha = g^2/4\pi$. The lowest-order Feynman diagrams contributing to fermion-antifermion annihilation into a pair of tensor gauge bosons are shown in Fig.3. In order g^2 , there are three diagrams. Dirac fermions ψ are conventionally drawn as thin solid lines, and Rarita-Schwinger spin-vector fermions ψ^μ by thick solid lines. These diagrams are similar to the QCD diagrams for fermion-antifermion annihilation into a pair of vector gauge bosons. The difference between these processes is in the actual expressions for the corresponding interaction vertices. Thus the probability amplitude of the process can be written in the form

$$i\mathcal{M}^{\mu\alpha\nu\beta} = (ig)^2 \bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{i\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{i\frac{1}{4}g^{\beta\alpha}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu - f^{abc} F^{\mu\alpha\rho\nu\beta} \gamma_\rho t^c \frac{1}{k_3^2} \right\} u(p_-)$$

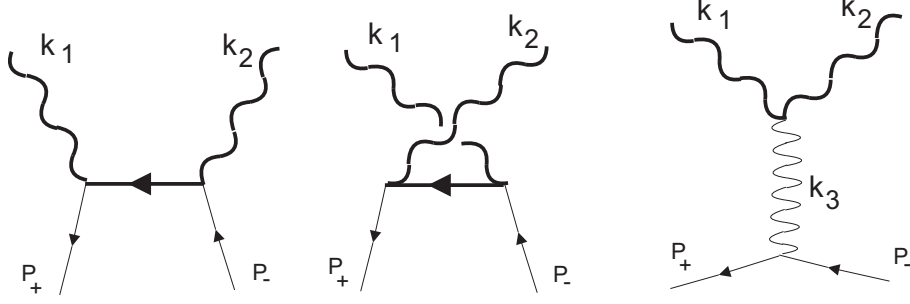


Figure 3: Diagrams contributing to fermion-antifermion annihilation to two tensor gauge bosons. Dirac fermions are conventionally drawn as thin solid lines, and Rarita-Schwinger spin-vector fermions by thick solid lines.

or in equivalent form as

$$\mathcal{M}^{\mu\alpha\nu\beta} = (ig)^2 \bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta} \right\} u(p_-) \quad (2.7)$$

where $u(p_-)$ is the wave function of spin 1/2 fermion and $v(p_+)$ of antifermion. The Dirac and symmetry group indices are not shown.

3 Gauge Invariance

Let us check that the amplitude is gauge invariant, that is, if we take the physical - transverse polarization - wave function e^T for one of the tensor gauge bosons and longitudinal polarization for the second one e^L , the transition amplitude vanishes $\mathcal{M}e^T e^L = 0$. This Ward identity expresses the fact that the unphysical - longitudinal polarization - states are not produced in the scattering process.

Contracting the matrix element (2.7) with the on-shell polarization tensors of the final tensor gauge bosons $e_{\mu\alpha}^*(k_1)$ and $e_{\nu\beta}^*(k_2)$ we shall get

$$\mathcal{M}^{\mu\alpha\nu\beta} e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) = (ig)^2 \bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta} \right\} u(p_-) e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2). \quad (3.8)$$

Considering the last term

$$i f^{abc} t^c \bar{v}(p_+) \gamma_\rho u(p_-) \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta}(k_1, k_3, k_2) e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2)$$

and taking the polarization tensor $e_{\nu\beta}^*(k_2)$ to be longitudinal

$$e_{\nu\beta}^*(k_2) = k_{2\nu}\xi_\beta + k_{2\beta}\xi_\nu$$

and the polarization tensor $e_{\mu\beta}^*(k_1)$ to be transversal, we shall get

$$\begin{aligned} & i f^{abc} t^c \bar{v}(p_+) \gamma_\rho u(p_-) \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta}(k_1, k_3, k_2) e_{\mu\alpha}^*(k_1) (k_{2\nu}\xi_\beta + k_{2\beta}\xi_\nu) = \\ & = i f^{abc} t^c \bar{v}(p_+) \gamma^\rho u(p_-) \frac{1}{k_3^2} [k_3^2 e_{\rho\alpha}^*(k_1) (\xi^\alpha - \frac{1}{2}\xi^\alpha) + k_3^2 e_{\alpha\rho}^*(k_1) (-\frac{1}{2}\xi^\alpha + \frac{1}{4}\xi^\alpha)] \end{aligned}$$

and therefore

$$\begin{aligned} & i f^{abc} t^c \bar{v}(p_+) \gamma_\rho u(p_-) \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta}(k_1, k_3, k_2) e_{\mu\alpha}^*(k_1) (k_{2\nu}\xi_\beta + k_{2\beta}\eta_\nu) = \\ & = i f^{abc} t^c \bar{v}(p_+) \gamma^\rho u(p_-) \frac{1}{4} e_{\rho\alpha}^*(k_1) \xi^\alpha. \end{aligned} \quad (3.9)$$

Now let us consider the first two terms

$$\bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu \right\} u(p_-) e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2).$$

Taking again the polarization tensor to be longitudinal $e_{\nu\beta}^*(k_2) = k_{2\nu}\xi_\beta + k_{2\beta}\xi_\nu$ we shall get

$$\begin{aligned} & \bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu \right\} u(p_-) e_{\mu\alpha}^*(k_1) (k_{2\nu}\xi_\beta + k_{2\beta}\xi_\nu) = \\ & = \frac{1}{4} \bar{v}(p_+) \left\{ -t^a t^b \gamma^\mu + t^b t^a \gamma^\mu \right\} u(p_-) e_{\mu\alpha}^*(k_1) g^{\alpha\beta} \xi_\beta + \\ & + \frac{1}{4} \bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{1}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{1}{\not{p}_- - \not{k}_1} t^a \gamma^\mu \right\} u(p_-) e_{\mu\alpha}^*(k_1) g^{\alpha\beta} k_{2\beta} \xi_\nu. \end{aligned}$$

The last term is equal to zero, $e_{\mu\alpha}^*(k_1) g^{\alpha\beta} k_{2\beta} = 0$, see relations (4.11), therefore we have

$$- \frac{1}{4} i f^{abc} t^c \bar{v}(p_+) \gamma^\mu u(p_-) e_{\mu\alpha}^*(k_1) \xi^\alpha. \quad (3.10)$$

This term precisely cancels the contribution coming from the last term of the amplitude (3.9). Thus the cross term matrix element between transverse and longitudinal polarizations vanishes $\mathcal{M} e^T e^L = 0$. Our intention now is to calculate *physical matrix element* with two transversal tensor gauge bosons $\mathcal{M} e^T e^T$ in the final state.

4 Squared Matrix Element

The complex conjugate of the scattering amplitude (2.7) is

$$\mathcal{M}^{*\mu\alpha\nu\beta} = (-ig)^2 \bar{u}(p_-) \left\{ \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^a \gamma^\mu + \gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^b \gamma^\nu - i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta} \right\} v(p_+)$$

and we can calculate now the squared matrix elements in the form

$$\begin{aligned} \mathcal{M}^{\mu\alpha\nu\beta} \mathcal{M}^{*\mu' \alpha' \nu' \beta'} &= (ig)^2 \bar{v}(p_+) \left\{ \gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + \right. \\ &\quad \left. + i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta} \right\} u(p_-) \\ &\quad * (-ig)^2 \bar{u}(p_-) \left\{ \gamma^{\nu'} t^{b'} \frac{\frac{1}{4}g^{\alpha'\beta'}}{\not{p}_- - \not{k}_2} t^{a'} \gamma^{\mu'} + \gamma^{\mu'} t^{a'} \frac{\frac{1}{4}g^{\alpha'\beta'}}{\not{p}_- - \not{k}_1} t^{b'} \gamma^{\nu'} - \right. \\ &\quad \left. - i f^{a'b'c'} t^{c'} \gamma_{\rho'} \frac{1}{k_3^2} F^{\mu' \alpha' \rho' \nu' \beta'} \right\} v(p_+) \end{aligned}$$

For unpolarized fermions-antifermion scattering the average over the fermion and antifermion spins is defined as follows:

$$|\mathcal{M}|^2 = \frac{1}{2} \frac{1}{2} \sum_{spin} |M|^2,$$

using completeness relations

$$\sum_s u^s(p_-) \bar{u}^s(p_-) = \not{p}_- , \quad \sum_s v^s(p_+) \bar{v}^s(p_+) = \not{p}_+ .$$

Thus averaging over spins of the fermions we shall get

$$\begin{aligned} \mathcal{M}^{\mu\alpha\nu\beta} \mathcal{M}^{*\mu' \alpha' \nu' \beta'} &= \frac{g^4}{4} Tr \left\{ \not{p}_+ \left[\gamma^\mu t^a \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4}g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + \right. \right. \\ &\quad \left. + i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta} \right] \\ &\quad \not{p}_- \left[\gamma^{\nu'} t^{b'} \frac{\frac{1}{4}g^{\alpha'\beta'}}{\not{p}_- - \not{k}_2} t^{a'} \gamma^{\mu'} + \gamma^{\mu'} t^{a'} \frac{\frac{1}{4}g^{\alpha'\beta'}}{\not{p}_- - \not{k}_1} t^{b'} \gamma^{\nu'} - \right. \\ &\quad \left. - i f^{a'b'c'} t^{c'} \gamma_{\rho'} \frac{1}{k_3^2} F^{\mu' \alpha' \rho' \nu' \beta'} \right] \left. \right\}. \end{aligned}$$

Contracting the last expression with the transversal on-shell polarization tensors of the final tensor gauge bosons $e_{\mu\alpha}^*(k_1)$ and $e_{\nu\beta}^*(k_2)$ we shall get the probability amplitude in the form

$$\mathcal{M}^{\mu\alpha\nu\beta} \mathcal{M}^{*\mu' \alpha' \nu' \beta'} e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) e_{\mu' \alpha'}(k_1) e_{\nu' \beta'}(k_2) =$$

$$\begin{aligned}
&= \frac{g^4}{4} Tr \{ \not{p}_+ [\gamma^\mu t^a \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} F^{\mu\alpha\rho\nu\beta}] \\
&\quad \not{p}_- [\gamma^{\nu'} t^{b'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_2} t^{a'} \gamma^{\mu'} + \gamma^{\mu'} t^{a'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_1} t^{b'} \gamma^{\nu'} - i f^{a'b'c'} t^{c'} \gamma_{\rho'} \frac{1}{k_3^2} F^{\mu'\alpha'\rho'\nu'\beta'}] \} \\
&\quad e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) e_{\mu'\alpha'}(k_1) e_{\nu'\beta'}(k_2).
\end{aligned}$$

Using the explicit form of the vertex operator $F^{\mu\alpha\rho\nu\beta}$ (2.3), (2.4) and the orthogonality properties of the tensor gauge boson wave functions

$$k_1^\mu e_{\mu\alpha}(k_1) = k_1^\alpha e_{\mu\alpha}(k_1) = k_2^\mu e_{\mu\alpha}(k_1) = k_2^\alpha e_{\mu\alpha}(k_1) = 0, \quad (4.11)$$

$$k_2^\mu e_{\mu\alpha}(k_2) = k_2^\alpha e_{\mu\alpha}(k_2) = k_1^\mu e_{\mu\alpha}(k_2) = k_1^\alpha e_{\mu\alpha}(k_2) = 0,$$

where the last relations follow from the fact that $\vec{k}_1 \parallel \vec{k}_2$ in the process of Fig.1, we shall get

$$\begin{aligned}
&\mathcal{M}^{\mu\alpha\nu\beta} \mathcal{M}^{*\mu'\alpha'\nu'\beta'} e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) e_{\mu'\alpha'}(k_1) e_{\nu'\beta'}(k_2) = \\
&= \frac{g^4}{4} Tr \{ \not{p}_+ [\gamma^\mu t^a \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + i f^{abc} t^c \gamma_\rho \frac{1}{k_3^2} (k_2 - k_1)^\rho (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha})] \\
&\quad \not{p}_- [\gamma^{\nu'} t^{b'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_2} t^{a'} \gamma^{\mu'} + \gamma^{\mu'} t^{a'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_1} t^{b'} \gamma^{\nu'} - i f^{a'b'c'} t^{c'} \gamma_{\rho'} \frac{1}{k_3^2} (k_2 - k_1)^{\rho'} (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'})] \} \\
&\quad e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) e_{\mu'\alpha'}(k_1) e_{\nu'\beta'}(k_2).
\end{aligned}$$

As the next step we shall calculate the sum over transversal tensor gauge bosons polarizations. The sum over transversal polarizations of the helicity-two tensor gauge boson is [4, 11]

$$\begin{aligned}
\sum e_{\mu\alpha}^*(k_1) e_{\mu'\alpha'}(k_1) &= \frac{1}{2} [(-g_{\mu\mu'} + \frac{k_{1\mu} \tilde{k}_{1\mu'} + \tilde{k}_{1\mu} k_{1\mu'}}{k_1 \tilde{k}_1}) (-g_{\alpha\alpha'} + \frac{k_{1\alpha} \tilde{k}_{1\alpha'} + \tilde{k}_{1\alpha} k_{1\alpha'}}{k_1 \tilde{k}_1}) + \\
&\quad + (-g_{\mu\alpha'} + \frac{k_{1\mu} \tilde{k}_{1\alpha'} + \tilde{k}_{1\mu} k_{1\alpha'}}{k_1 \tilde{k}_1}) (-g_{\alpha\mu'} + \frac{k_{1\alpha} \tilde{k}_{1\mu'} + \tilde{k}_{1\alpha} k_{1\mu'}}{k_1 \tilde{k}_1}) - \\
&\quad - (-g_{\mu\alpha} + \frac{k_{1\mu} \tilde{k}_{1\alpha} + \tilde{k}_{1\mu} k_{1\alpha}}{k_1 \tilde{k}_1}) (-g_{\mu'\alpha'} + \frac{k_{1\mu'} \tilde{k}_{1\alpha'} + \tilde{k}_{1\mu'} k_{1\alpha'}}{k_1 \tilde{k}_1})],
\end{aligned}$$

where $k_{1\mu} = (\omega_1, \vec{k}_1)$ and $\tilde{k}_{1\mu} = (\omega_1, -\vec{k}_1)$. The explicit form of the transversal polarization tensors, when momentum is aligned along the third axis, is given by the matrices [4, 11]

$$e_{\mu\alpha}^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0, 0, & 0, 0 \\ 0, 1, & 0, 0 \\ 0, 0, & -1, 0 \\ 0, 0, & 0, 0 \end{pmatrix}, e_{\mu\alpha}^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0, 0, 0, 0 \\ 0, 0, 1, 0 \\ 0, 1, 0, 0 \\ 0, 0, 0, 0 \end{pmatrix}.$$

From the kinematics of the process in Fig.1 it follows that $\omega_2 = \omega_1$ and $\vec{k}_2 = -\vec{k}_1$ therefore

$$\tilde{k}_{1\mu} = k_{2\mu}, \quad \tilde{k}_{2\mu} = k_{1\mu}$$

and the average over polarizations can be rewritten as

$$\sum e_{\mu\alpha}^*(k_1) e_{\mu'\alpha'}(k_1) = \frac{1}{2}(E_{\mu\mu'} E_{\alpha\alpha'} + E_{\mu\alpha'} E_{\alpha\mu'} - E_{\mu\alpha} E_{\mu'\alpha'}), \quad (4.12)$$

where

$$E_{\mu\mu'} = -g_{\mu\mu'} + \frac{k_{1\mu} k_{2\mu'} + k_{2\mu} k_{1\mu'}}{k_1 \cdot k_2}.$$

Thus the average over tensor gauge boson polarizations gives

$$\begin{aligned} & \mathcal{M}^{\mu\alpha\nu\beta} \mathcal{M}^{*\mu'\alpha'\nu'\beta'} \sum e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) \sum e_{\mu'\alpha'}(k_1) e_{\nu'\beta'}(k_2) = \\ &= \frac{g^4}{4} Tr \{ \not{p}_+ [\gamma^\mu t^a \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu + \gamma^\nu t^b \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_1} t^a \gamma^\mu + i f^{abc} t^c \frac{1}{k_3^2} (\not{k}_2 - \not{k}_1) (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha})] \\ & \not{p}_- [\gamma^{\nu'} t^{b'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_2} t^{a'} \gamma^{\mu'} + \gamma^{\mu'} t^{a'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_1} t^{b'} \gamma^{\nu'} - i f^{a'b'c'} t^{c'} \frac{1}{k_3^2} (\not{k}_2 - \not{k}_1) (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'})] \} \\ & \frac{\delta^{aa'}}{d(r)} \frac{\delta^{bb'}}{d(r)} \frac{1}{2} (E_{\mu\mu'} E_{\alpha\alpha'} + E_{\mu\alpha'} E_{\alpha\mu'} - E_{\mu\alpha} E_{\mu'\alpha'}) \frac{1}{2} (E_{\nu\nu'} E_{\beta\beta'} + E_{\nu\beta'} E_{\beta\nu'} - E_{\nu\beta} E_{\nu'\beta'}). \end{aligned} \quad (4.13)$$

In the next section we shall evaluate these traces and summation over polarizations.

5 Evaluation of Traces

In order to evaluate the squared matrix element in the last expression (4.13) we have to calculate traces and then perform summation over polarizations. We shall use convenient notations for the different terms in the amplitude. The whole amplitude will be expressed as a symbolic sum of three terms

$$\mathcal{M} = R + L + G,$$

exactly corresponding to the three Feynman diagrams in Fig.3, so that the squared amplitude (4.13) shall have nine terms

$$\mathcal{M}\mathcal{M}^* = (R + L + G)(R + L + G)^*.$$

The first contribution can be evaluated in the following way:

$$\begin{aligned}
(GG^*)^{\mu\alpha\nu\beta \ \mu' \alpha' \nu' \beta'} &= \frac{g^4}{4d^2(r)} Tr\{\not{p}_+ i f^{abc} t^c \frac{1}{k_3^2} (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha}) (\not{k}_2 - \not{k}_1) \\
&\quad \not{p}_- (-i) f^{a'b'c'} t^{c'} \frac{1}{k_3^2} (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}) (\not{k}_2 - \not{k}_1)\} \delta^{aa'} \delta^{bb'} = \\
&= \frac{g^4}{4d^2(r)} tr(f^{abc} f^{abc'} t^c t^{c'}) \frac{Tr\{\not{p}_+ (\not{k}_2 - \not{k}_1) \not{p}_- (\not{k}_2 - \not{k}_1)\}}{(2k_1 k_2)(2k_1 k_2)} \\
&\quad (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha}) (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}).
\end{aligned}$$

We can calculate traces over the symmetry group indices using formulas from the Appendix A:

$$tr(f^{abc} f^{abc'} t^c t^{c'}) = d(r) C_2(r) C_2(G) = d(G) C(r) C_2(G) = \frac{N(N^2 - 1)}{2}$$

and then the traces of gamma matrices using relation from the Appendix B:

$$\begin{aligned}
&\frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) 8 \frac{p_+ \cdot (k_2 - k_1) p_- \cdot (k_2 - k_1) + p_+ \cdot p_- k_1 \cdot k_2}{(2k_1 k_2)(2k_1 k_2)} \\
&\quad (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha}) (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}) = \\
&= \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) 2 \sin^2 \theta (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha}) (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}).
\end{aligned}$$

Now it is easy to calculate summation over tensor gauge boson polarizations using expression (4.12) and the corresponding scalar products (2.2)

$$GG^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \sin^2 \theta. \quad (5.14)$$

The next contribution in (4.13) can be evaluated as follows:

$$\begin{aligned}
(LG^*)^{\mu\alpha\nu\beta \ \mu' \alpha' \nu' \beta'} &= \frac{g^4}{4d^2(r)} Tr\{\not{p}_+ [\gamma^\mu t^a \frac{1}{4} g^{\alpha\beta} t^b \gamma^\nu] \\
&\quad \not{p}_- [-i f^{a'b'c'} t^{c'} \frac{1}{k_3^2} (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}) (\not{k}_2 - \not{k}_1)]\} \delta^{aa'} \delta^{bb'} = \\
&= -i \frac{g^4}{4d^2(r)} tr(f^{abc} t^a t^b t^c) \frac{Tr\{\not{p}_+ \gamma^\mu (\not{p}_- - \not{k}_2) \gamma^\nu \not{p}_- (\not{k}_2 - \not{k}_1)\}}{(-2p_- k_2)(2k_1 k_2)} \frac{1}{4} g^{\alpha\beta} (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}),
\end{aligned}$$

and then using traces from the Appendix A and the Appendix B we shall get

$$\begin{aligned}
&\frac{g^4}{4d^2(r)} \frac{d(r) C_2(r) C_2(G)}{2} 4 \{g^{\mu\nu} [p_+ \cdot p_- k_1 \cdot k_2 + p_+ \cdot (k_2 - k_1) p_- \cdot k_2 + p_+ \cdot k_2 p_- \cdot (k_2 - k_1)] + \\
&\quad + k_1 \cdot k_2 (p_+^\mu p_-^\nu - p_-^\mu p_+^\nu) + 2p_- \cdot (k_2 - k_1) p_+^\mu p_-^\nu + 2p_+ \cdot (k_2 - k_1) p_-^\mu p_+^\nu\} \\
&\quad \frac{1}{(-2p_- k_2)(2k_1 k_2)} \frac{1}{4} g^{\alpha\beta} (g^{\mu'\nu'} g^{\alpha'\beta'} - \frac{1}{2} g^{\mu'\beta'} g^{\nu'\alpha'}).
\end{aligned}$$

Using again expression (4.12) and scalar products (2.2) we can sum over the polarizations of tensor gauge bosons:

$$LG^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \left(-\frac{1}{4} \sin^2 \theta\right). \quad (5.15)$$

The third contribution is

$$\begin{aligned} (RG^*)^{\mu\alpha\nu\beta \ \mu' \alpha' \nu' \beta'} &= \frac{g^4}{4d^2(r)} \text{Tr} \left\{ \not{p}_+ [\gamma^\nu t^b \frac{1}{4} g^{\alpha\beta} t^a \gamma^\mu] \right. \\ &\quad \left. \not{p}_- [-i f^{a' b' c' t' c} \frac{1}{k_3^2} (g^{\mu' \nu'} g^{\alpha' \beta'} - \frac{1}{2} g^{\mu' \beta'} g^{\nu' \alpha'}) (\not{k}_2 - \not{k}_1)] \right\} \delta^{aa'} \delta^{bb'} = \\ &= -i \frac{g^4}{4d^2(r)} \text{tr}(f^{abc} t^b t^a t^c) \frac{\text{Tr} \{ \not{p}_+ \gamma^\nu (\not{p}_- - \not{k}_1) \gamma^\mu \not{p}_- (\not{k}_2 - \not{k}_1) \}}{(-2p_- k_1)(2k_1 k_2)} \frac{1}{4} g^{\alpha\beta} (g^{\mu' \nu'} g^{\alpha' \beta'} - \frac{1}{2} g^{\mu' \beta'} g^{\nu' \alpha'}) \end{aligned}$$

and can be evaluated in the similar way:

$$\begin{aligned} \frac{g^4}{4d^2(r)} &\left(-\frac{d(r) C_2(r) C_2(G)}{2} \right) 4 \{ g^{\nu\mu} [-p_+ \cdot p_- \not{k}_1 \cdot k_2 + p_+ \cdot (k_2 - k_1) \not{p}_- \cdot k_1 + p_+ \cdot k_1 \not{p}_- \cdot (k_2 - k_1)] + \\ &\quad + k_1 \cdot k_2 (p_-^\nu p_+^\mu - p_+^\nu p_-^\mu) + 2p_- \cdot (k_2 - k_1) p_+^\nu p_-^\mu + 2p_+ \cdot (k_2 - k_1) p_-^\nu p_-^\mu \} \frac{1}{(-2p_- k_1)(2k_1 k_2)} \\ &\quad \frac{1}{4} g^{\alpha\beta} (g^{\mu' \nu'} g^{\alpha' \beta'} - \frac{1}{2} g^{\mu' \beta'} g^{\nu' \alpha'}), \end{aligned}$$

so that after summation over polarization we shall get:

$$(RG^*) = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \left(-\frac{1}{4} \sin^2 \theta\right). \quad (5.16)$$

As one can get convinced, the next two terms GL^* and GR^* give similar contributions:

$$GL^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \left(-\frac{1}{4} \sin^2 \theta\right), \quad (5.17)$$

$$GR^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \left(-\frac{1}{4} \sin^2 \theta\right). \quad (5.18)$$

The sixth contribution is

$$\begin{aligned} (LL^*)^{\mu\alpha\nu\beta \ \mu' \alpha' \nu' \beta'} &= \frac{g^4}{4d^2(r)} \text{Tr} \left\{ \not{p}_+ [\gamma^\mu t^a \frac{1}{4} g^{\alpha\beta} t^b \gamma^\nu] \not{p}_- [\gamma^{\nu'} t^{b'} \frac{1}{4} g^{\alpha' \beta'} t^{a'} \gamma^{\mu'}] \right\} \delta^{aa'} \delta^{bb'} = \\ &= \frac{g^4}{4d^2(r)} \text{tr}(t^a t^b t^a t^b) \frac{\text{Tr} \{ \not{p}_+ \gamma^\mu (\not{p}_- - \not{k}_2) \gamma^\nu \not{p}_- \gamma^{\nu'} (\not{p}_- - \not{k}_2) \gamma^{\mu'} \}}{(2p_- k_2)(2p_- k_2)} \frac{1}{4} g^{\alpha\beta} \frac{1}{4} g^{\alpha' \beta'} \end{aligned}$$

and involves trace of eight gamma matrices. It can be performed using expression presented in the Appendix B:

$$\begin{aligned} & \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(r) \{ 16 p_-^\nu p_-^{\nu'} [p_-^\mu p_+^{\mu'} + p_+^\mu p_-^{\mu'} - p_+ \cdot p_- g^{\mu\mu'}] + \\ & + 8 p_-^{\nu'} [(p_+ \cdot k_2 p_-^{\mu'} + p_- \cdot k_2 p_+^{\mu'}) g^{\mu\nu} - (p_+ \cdot k_2 p_-^\mu - p_- \cdot k_2 p_+^\mu) g^{\mu'\nu} + (p_+ \cdot k_2 p_-^\nu - p_- \cdot k_2 p_+^\nu) g^{\mu\mu'}] \\ & + 8 p_-^\nu [(p_+ \cdot k_2 p_-^\mu + p_- \cdot k_2 p_+^\mu) g^{\mu'\nu'} - (p_+ \cdot k_2 p_-^{\mu'} - p_- \cdot k_2 p_+^{\mu'}) g^{\mu\nu'} + (p_+ \cdot k_2 p_-^{\nu'} - p_- \cdot k_2 p_+^{\nu'}) g^{\mu\mu'}] \\ & + 8 p_+ \cdot k_2 p_- \cdot k_2 [g^{\mu\nu} g^{\mu'\nu'} - g^{\mu\nu'} g^{\mu'\nu} + g^{\mu\mu'} g^{\nu\nu'}] \} \frac{1}{(2p_- k_2)(2p_- k_2)} \frac{1}{4} g^{\alpha\beta} \frac{1}{4} g^{\alpha'\beta'}, \end{aligned}$$

and after summation over polarizations we shall get

$$LL^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(r) \frac{1}{4} \sin^2 \theta. \quad (5.19)$$

The seventh contribution is identical with the sixth one and gives

$$RR^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(r) \frac{1}{4} \sin^2 \theta. \quad (5.20)$$

The eighth contribution is

$$\begin{aligned} (LR^*)^{\mu\alpha\nu\beta \mu' \alpha' \nu' \beta'} &= \frac{g^4}{4d^2(r)} Tr \{ \not{p}_+ [\gamma^\mu t^a \frac{\frac{1}{4} g^{\alpha\beta}}{\not{p}_- - \not{k}_2} t^b \gamma^\nu] \not{p}_- [\gamma^{\mu'} t^{a'} \frac{\frac{1}{4} g^{\alpha'\beta'}}{\not{p}_- - \not{k}_1} t^{b'} \gamma^{\nu'}] \} \delta^{aa'} \delta^{bb'} = \\ &= \frac{g^4}{4d^2(r)} tr(t^a t^b t^{a'} t^{b'}) \frac{Tr \{ \not{p}_+ \gamma^\mu (\not{p}_- - \not{k}_2) \gamma^\nu \not{p}_- \gamma^{\mu'} (\not{p}_- - \not{k}_1) \gamma^{\nu'} \} \frac{1}{4} g^{\alpha\beta} \frac{1}{4} g^{\alpha'\beta'}}{(2p_- k_2)(2p_- k_1)} \end{aligned}$$

and gives

$$\begin{aligned} & \frac{g^4}{4d^2(r)} d(r) C_2(r) (C_2(r) - \frac{1}{2} C_2(G)) \{ -4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\nu'} g^{\nu\mu'} + 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\nu'} g^{\nu\mu'} + \\ & + 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\nu'} g^{\nu\mu'} - 4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\nu} g^{\mu'\nu'} + \\ & + 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\nu} g^{\mu'\nu'} + 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\nu} g^{\mu'\nu'} + \\ & + 4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\mu'} g^{\nu\nu'} - 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\mu'} g^{\nu\nu'} - \\ & - 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\mu'} g^{\nu\nu'} - 4k_1 \cdot k_2 g^{\nu\nu'} p_+^\mu p_-^\mu + 4k_1 \cdot k_2 g^{\mu'\nu'} p_+^\nu p_-^\nu + 4k_1 \cdot k_2 g^{\nu\mu'} p_+^\nu p_-^\nu - \\ & - 4k_1 \cdot k_2 g^{\nu\nu'} p_+^\mu p_-^\mu + 8p_- \cdot k_2 g^{\nu\nu'} p_+^\mu p_-^\mu + 4k_1 \cdot k_2 g^{\mu\nu'} p_+^\nu p_-^\nu - 8p_- \cdot k_2 g^{\mu\nu'} p_+^\nu p_-^\nu - \\ & - 4k_1 \cdot k_2 g^{\mu\nu} p_+^\nu p_-^\mu + 8p_- \cdot k_2 g^{\mu\nu} p_+^\nu p_-^\mu - 8p_+ \cdot k_2 g^{\nu\nu'} p_-^\mu p_-^\mu - 4k_1 \cdot k_2 g^{\mu'\nu'} p_+^\mu p_-^\nu + \\ & + 8p_- \cdot k_1 g^{\mu'\nu'} p_+^\mu p_-^\nu + 4k_1 \cdot k_2 g^{\mu\nu'} p_+^\mu p_-^\nu - 8p_- \cdot k_1 g^{\mu\nu'} p_+^\mu p_-^\nu - 4k_1 \cdot k_2 g^{\mu\mu'} p_+^\nu p_-^\nu + \\ & + 8p_- \cdot k_1 g^{\mu\mu'} p_+^\nu p_-^\nu + 8p_+ \cdot k_1 g^{\mu'\nu'} p_-^\mu p_-^\nu + 8p_+ \cdot k_1 g^{\mu\nu'} p_-^\mu p_-^\nu + 8p_+ \cdot k_2 g^{\mu\nu'} p_-^\mu p_-^\nu - \\ & - 16p_+ \cdot p_- g^{\mu\nu'} p_-^\mu p_-^\nu + 16p_+^\nu p_-^\mu p_-^\mu p_-^\nu + 4k_1 \cdot k_2 g^{\nu\mu'} p_+^\mu p_-^\nu + 4k_1 \cdot k_2 g^{\mu\nu} p_+^\mu p_-^\nu - \end{aligned}$$

$$-4k_1 \cdot k_2 g^{\mu\mu'} p_+^\nu p_-^{\nu'} + 8p_+ \cdot k_2 g^{\mu\nu} p_-^{\mu'} p_-^{\nu'} - 8p_+ \cdot k_1 g^{\mu\mu'} p_-^\nu p_-^{\nu'} + 16p_+^\mu p_-^{\mu'} p_-^\nu p_-^{\nu'} \} \\ \frac{1}{(2p_- k_2)(2p_- k_1)} \frac{1}{4} g^{\alpha\beta} \frac{1}{4} g^{\alpha'\beta'}.$$

After summation over polarizations we shall get

$$LR^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) (C_2(r) - \frac{1}{2} C_2(G)) \left(-\frac{1}{4} \sin^2 \theta\right) \quad (5.21)$$

and for the ninth contribution we shall get identically

$$RL^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) (C_2(r) - \frac{1}{2} C_2(G)) \left(-\frac{1}{4} \sin^2 \theta\right). \quad (5.22)$$

We are now in a position to calculate the total contribution to the squared matrix element (4.13). Putting together all pieces of the squared matrix element (5.14), (5.15), (5.16), (5.17), (5.18), (5.19), (5.20), (5.21), (5.22), we finally obtain

$$\mathcal{M}\mathcal{M}^* = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \frac{1}{4} \sin^2 \theta. \quad (5.23)$$

6 Cross Section

We can calculate now the leading-order differential cross section for the tensor gauge bosons production in the annihilation process. This process, as we discussed in the introduction, receives contribution from three Feynman diagrams shown in Fig.3 and for the unpolarized fermion pairs the squared matrix element was presented above (5.23). Plugging everything into our general cross-section formula (2.6) yields the differential cross section in the center-of-mass frame:

$$d\sigma = \frac{g^4}{4d^2(r)} d(r) C_2(r) C_2(G) \frac{1}{4} \sin^2 \theta \frac{1}{2s} \frac{1}{32\pi^2} d\Omega = \\ = \left(\frac{g^2}{4\pi}\right)^2 \frac{1}{s} \frac{C_2(r) C_2(G)}{64d(r)} \sin^2 \theta d\Omega = \\ = \frac{\alpha^2}{s} \frac{C_2(r) C_2(G)}{64d(r)} \sin^2 \theta d\Omega, \quad (6.24)$$

where

$$\alpha = \frac{g^2}{4\pi}.$$

Thus the unpolarized cross section is

$$d\sigma = \frac{\alpha^2}{s} \frac{C_2(r) C_2(G)}{64d(r)} \sin^2 \theta d\Omega, \quad (6.25)$$

where for the $SU(N)$ group we have $\frac{C_2(r)C_2(G)}{64d(r)} = \frac{(N^2-1)}{128N}$. This cross section should be compared with the analogous annihilation cross sections in QED and QCD. Indeed let us compare this result with the electron-positron annihilation into two photons. The $e^+e^- \rightarrow \gamma\gamma$ annihilation cross section [12] in the high-energy limit is

$$d\sigma_{\gamma\gamma} = \frac{\alpha^2}{s} \frac{1 + \cos^2 \theta}{\sin^2 \theta} d\Omega \quad (6.26)$$

except very small angles of order m/E . Angular dependence of cross section is such that at $\theta = \pi/2$ it has a minimum and then increases for small angles [13, 14]. The quark pair annihilation cross section into two gluons $q\bar{q} \rightarrow gg$ in the leading order of the strong coupling α_s is

$$d\sigma_{gg} = \frac{\alpha_s^2}{s} \frac{C_2(r)C_2(r)}{d(r)} \left[\frac{1 + \cos^2 \theta}{\sin^2 \theta} - \frac{C_2(G)}{4C_2(r)} (1 + \cos^2 \theta) \right] d\Omega \quad (6.27)$$

and also has minimum at $\theta = \pi/2$ and increases for small scattering angles [15]. The production cross section of tensor gauge bosons (6.25) shows dramatically different behavior with its maximum at $\theta = \pi/2$ and decrease for small angles.

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7 Appendix A

The gauge group matrices t^a form a representation r of the Lie group G . The matrices t^a obey the algebra $[t^a, t^b] = if^{abc}t^c$, where the structure constants f^{abc} are totally antisymmetric. The invariant operators $C(r)$ and $C_2(r)$ are defined by the equations

$$t^a t^b = C_2(r)1, \quad tr(t^a t^b) = C(r)\delta^{ab}$$

and satisfy the relation

$$C(r) = \frac{d(r)}{d(G)} C_2(r),$$

where $d(r)$ is the dimension of the representation r . By convention the i and a are indices of the symmetry group G . A number of fermions ψ^i is equal to the dimension $d(r)$ of the

representation $r: i = 1, \dots, d(r)$. The number of gauge bosons A^a is equal to the number $d(G)$ of generators of the group $G: a = 1, \dots, d(G)$. For the fundamental N and adjoint G representations of the $SU(N)$ groups we have

$$C(N) = 1/2, \quad C_2(N) = \frac{N^2 - 1}{2N}, \quad C(G) = 1/2 = C_2(G) = N.$$

The traces over symmetry group indices now can be evaluated:

$$\begin{aligned} -itr(f^{abc}t^at^bt^c) &= \frac{1}{2}d(r)C_2(r)C_2(G) = \frac{1}{2}d(G)C(r)C_2(G) = \frac{N(N^2 - 1)}{4}, \\ tr(f^{abc}f^{abc'}t^ct^{c'}) &= d(r)C_2(r)C_2(G) = d(G)C(r)C_2(G) = \frac{N(N^2 - 1)}{2}, \\ tr(t^at^bt^bt^a) &= d(r)C_2(r)C_2(r) = d(G)C(r)C_2(r) = \frac{(N^2 - 1)^2}{4N}, \\ tr(t^at^bt^at^b) &= d(G)C(r)C_2(r)(C_2(r) - \frac{1}{2}C_2(G)) = -\frac{(N^2 - 1)}{4N}. \end{aligned}$$

8 Appendix B

In this appendix we shall perform calculation of traces which appear in the squared matrix element (4.13). The traces under consideration have terms proportional to the momentum of the tensor gauge bosons k_1^μ and k_2^ν . These terms can be ignored, because after contraction with the corresponding transverse wave functions of the tensor gauge bosons $e_{\mu\alpha}(k_1)$ and $e_{\nu\beta}(k_2)$ they give zero contribution. Therefore we shall calculate the traces up to the longitudinal terms which are proportional to the vectors k_1^μ and k_2^ν . They are

$$GG \sim Tr\{\not{p}_+(\not{k}_2 - \not{k}_1) \not{p}_-(\not{k}_2 - \not{k}_1)\} = 8[p_+ \cdot (k_2 - k_1) p_- \cdot (k_2 - k_1) + p_+ \cdot p_- k_1 \cdot k_2],$$

$$\begin{aligned} (LG^*)^{\mu\nu} &\sim Tr\{\not{p}_+\gamma^\mu(\not{p}_- - \not{k}_2)\gamma^\nu \not{p}_-(\not{k}_2 - \not{k}_1)\} = \\ &= 4\{g^{\mu\nu}[p_+ \cdot p_- k_1 \cdot k_2 + p_+ \cdot (k_2 - k_1) p_- \cdot k_2 + p_+ \cdot k_2 p_- \cdot (k_2 - k_1)] + \\ &+ k_1 \cdot k_2(p_+^\mu p_-^\nu - p_-^\mu p_+^\nu) + 2p_- \cdot (k_2 - k_1)p_+^\mu p_-^\nu + 2p_+ \cdot (k_2 - k_1)p_-^\mu p_-^\nu\}, \end{aligned}$$

$$\begin{aligned} (RG^*)^{\mu\nu} &\sim Tr\{\not{p}_+\gamma^\nu(\not{p}_- - \not{k}_1)\gamma^\mu \not{p}_-(\not{k}_2 - \not{k}_1)\} = \\ &= 4\{g^{\nu\mu}[-p_+ \cdot p_- k_1 \cdot k_2 + p_+ \cdot (k_2 - k_1) p_- \cdot k_1 + p_+ \cdot k_1 p_- \cdot (k_2 - k_1)] + \\ &+ k_1 \cdot k_2(p_-^\nu p_+^\mu - p_+^\nu p_-^\mu) + 2p_- \cdot (k_2 - k_1)p_+^\nu p_-^\mu + 2p_+ \cdot (k_2 - k_1)p_-^\nu p_-^\mu\}, \end{aligned}$$

$$\begin{aligned}
(GL^*)^{\mu\nu} &\sim Tr\{\not{p}_+(\not{k}_2 - \not{k}_1) \not{p}_- \gamma^\nu (\not{p}_- - \not{k}_2) \gamma^\mu\} = \\
&= 4\{g^{\mu\nu}[p_+ \cdot p_- k_1 \cdot k_2 + p_+ \cdot (k_2 - k_1) p_- \cdot k_2 + p_+ \cdot k_2 p_- \cdot (k_2 - k_1)] + \\
&+ k_1 \cdot k_2 (p_+^\mu p_-^\nu - p_-^\mu p_+^\nu) + 2p_- \cdot (k_2 - k_1) p_+^\mu p_-^\nu + 2p_+ \cdot (k_2 - k_1) p_-^\mu p_-^\nu\},
\end{aligned}$$

$$\begin{aligned}
(GR^*)^{\mu\nu} &\sim Tr\{\not{p}_+(\not{k}_2 - \not{k}_1) \not{p}_- \gamma^\mu (\not{p}_- - \not{k}_1) \gamma^\nu\} = \\
&= 4\{g^{\mu\nu}[-p_+ \cdot p_- k_1 \cdot k_2 + p_+ \cdot (k_2 - k_1) p_- \cdot k_1 + p_+ \cdot k_1 p_- \cdot (k_2 - k_1)] + \\
&+ k_1 \cdot k_2 (p_+^\mu p_-^\nu - p_-^\mu p_+^\nu) + 2p_- \cdot (k_2 - k_1) p_+^\nu p_-^\mu + 2p_+ \cdot (k_2 - k_1) p_-^\mu p_-^\nu\},
\end{aligned}$$

$$\begin{aligned}
(LL^*)^{\mu\nu\mu'\nu'} &\sim Tr\{\not{p}_+ \gamma^\mu (\not{p}_- - \not{k}_2) \gamma^\nu \not{p}_- \gamma^{\mu'} (\not{p}_- - \not{k}_2) \gamma^{\nu'}\} = \\
&= \{16p_-^\nu p_-^{\nu'} [p_-^\mu p_+^{\mu'} + p_+^\mu p_-^{\mu'} - p_+ \cdot p_- g^{\mu\mu'}] + \\
&+ 8p_-^{\nu'} [(p_+ \cdot k_2 p_-^{\mu'} + p_- \cdot k_2 p_+^{\mu'}) g^{\mu\nu} - (p_+ \cdot k_2 p_-^\mu - p_- \cdot k_2 p_+^\mu) g^{\mu'\nu} + (p_+ \cdot k_2 p_-^\nu - p_- \cdot k_2 p_+^\nu) g^{\mu\mu'}] \\
&+ 8p_-^\nu [(p_+ \cdot k_2 p_-^\mu + p_- \cdot k_2 p_+^\mu) g^{\mu'\nu'} - (p_+ \cdot k_2 p_-^{\mu'} - p_- \cdot k_2 p_+^{\mu'}) g^{\mu\nu'} + (p_+ \cdot k_2 p_-^{\nu'} - p_- \cdot k_2 p_+^{\nu'}) g^{\mu\mu'}] \\
&+ 8p_+ \cdot k_2 p_- \cdot k_2 [g^{\mu\nu} g^{\mu'\nu'} - g^{\mu\nu'} g^{\mu'\nu} + g^{\mu\mu'} g^{\nu\nu'}]\},
\end{aligned}$$

$$\begin{aligned}
(RR^*)^{\mu\nu\mu'\nu'} &\sim Tr\{\not{p}_+ \gamma^\nu (\not{p}_- - \not{k}_1) \gamma^\mu \not{p}_- \gamma^{\mu'} (\not{p}_- - \not{k}_1) \gamma^{\nu'}\} = \\
&= \{16p_-^\mu p_-^{\mu'} [p_+^\nu p_-^{\nu'} + p_-^\nu p_+^{\nu'} - p_+ \cdot p_- g^{\nu\nu'}] + \\
&+ 8p_-^{\mu'} [(p_+ \cdot k_1 p_-^{\nu'} + p_- \cdot k_1 p_+^{\nu'}) g^{\nu\mu} - (p_+ \cdot k_1 p_-^\nu - p_- \cdot k_1 p_+^\nu) g^{\nu'\mu} + (p_+ \cdot k_1 p_-^\mu - p_- \cdot k_1 p_+^\mu) g^{\nu\nu'}] \\
&+ 8p_-^\mu [(p_+ \cdot k_1 p_-^\nu + p_- \cdot k_1 p_+^\nu) g^{\nu'\mu'} - (p_+ \cdot k_1 p_-^{\nu'} - p_- \cdot k_1 p_+^{\nu'}) g^{\nu\mu'} + (p_+ \cdot k_1 p_-^{\mu'} - p_- \cdot k_1 p_+^{\mu'}) g^{\nu\nu'}] \\
&+ 8p_+ \cdot k_1 p_- \cdot k_1 [g^{\nu\mu} g^{\mu'\nu'} - g^{\nu\mu'} g^{\mu\nu'} + g^{\nu\nu'} g^{\mu\mu'}]\},
\end{aligned}$$

$$\begin{aligned}
(LR^*)^{\mu\nu\mu'\nu'} &\sim Tr\{\not{p}_+ \gamma^\mu (\not{p}_- - \not{k}_2) \gamma^\nu \not{p}_- \gamma^{\mu'} (\not{p}_- - \not{k}_1) \gamma^{\nu'}\} = \\
&= -4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\nu'} g^{\nu\mu'} + 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\nu'} g^{\nu\mu'} + \\
&4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\nu'} g^{\nu\mu'} - 4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\nu} g^{\mu'\nu'} + \\
&4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\nu} g^{\mu'\nu'} + 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\nu} g^{\mu'\nu'} +
\end{aligned}$$

$$\begin{aligned}
& 4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\mu'} g^{\nu\nu'} - 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\mu'} g^{\nu\nu'} - \\
& 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\mu'} g^{\nu\nu'} - 4k_1 \cdot k_2 g^{\nu\nu'} p_+^{\mu'} p_-^{\mu} + \\
& 4k_1 \cdot k_2 g^{\mu'\nu'} p_+^{\nu} p_-^{\mu} + 4k_1 \cdot k_2 g^{\nu\mu'} p_+^{\nu'} p_-^{\mu} - \\
& 4k_1 \cdot k_2 g^{\nu\nu'} p_+^{\mu} p_-^{\mu'} + 8p_- \cdot k_2 g^{\nu\nu'} p_+^{\mu} p_-^{\mu'} + \\
& 4k_1 \cdot k_2 g^{\mu\nu'} p_+^{\nu} p_-^{\mu'} - 8p_- \cdot k_2 g^{\mu\nu'} p_+^{\nu} p_-^{\mu'} - \\
& 4k_1 \cdot k_2 g^{\mu\nu} p_+^{\nu'} p_-^{\mu'} + 8p_- \cdot k_2 g^{\mu\nu} p_+^{\nu'} p_-^{\mu'} - \\
& 8p_+ \cdot k_2 g^{\nu\nu'} p_-^{\mu} p_-^{\mu'} - 4k_1 \cdot k_2 g^{\mu'\nu'} p_+^{\mu} p_-^{\nu} + \\
& 8p_- \cdot k_1 g^{\mu'\nu'} p_+^{\mu} p_-^{\nu} + 4k_1 \cdot k_2 g^{\mu\nu'} p_+^{\mu'} p_-^{\nu} - \\
& 8p_- \cdot k_1 g^{\mu\nu'} p_+^{\mu'} p_-^{\nu} - 4k_1 \cdot k_2 g^{\mu\mu'} p_+^{\nu'} p_-^{\nu} + \\
& 8p_- \cdot k_1 g^{\mu\mu'} p_+^{\nu'} p_-^{\nu} + 8p_+ \cdot k_1 g^{\mu'\nu'} p_-^{\mu} p_-^{\nu} + \\
& 8p_+ \cdot k_1 g^{\mu\nu'} p_-^{\mu'} p_-^{\nu} + 8p_+ \cdot k_2 g^{\mu\nu'} p_-^{\mu'} p_-^{\nu} - \\
& 16p_+ \cdot p_- g^{\mu\nu'} p_-^{\mu'} p_-^{\nu} + 16p_+^{\nu'} p_-^{\mu'} p_-^{\nu} p_-^{\nu} + \\
& 4k_1 \cdot k_2 g^{\nu\mu'} p_+^{\mu} p_-^{\nu'} + 4k_1 \cdot k_2 g^{\mu\nu} p_+^{\mu'} p_-^{\nu'} - \\
& 4k_1 \cdot k_2 g^{\mu\mu'} p_+^{\nu} p_-^{\nu'} + 8p_+ \cdot k_2 g^{\mu\nu} p_-^{\mu'} p_-^{\nu'} - \\
& 8p_+ \cdot k_1 g^{\mu\mu'} p_-^{\nu} p_-^{\nu'} + 16p_+^{\mu} p_-^{\mu'} p_-^{\nu} p_-^{\nu'},
\end{aligned}$$

$$\begin{aligned}
& (RL^*)^{\mu\nu\mu'\nu'} \sim Tr\{\not{p}_+ \gamma^{\nu} (\not{p}_- - \not{k}_1) \gamma^{\mu} \not{p}_- \gamma^{\nu'} (\not{p}_- - \not{k}_2) \gamma^{\mu'}\} = \\
& = -4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\nu'} g^{\nu\mu'} + 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\nu'} g^{\nu\mu'} + \\
& 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\nu'} g^{\nu\mu'} - 4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\nu} g^{\mu'\nu'} + \\
& 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\nu} g^{\mu'\nu'} + 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\nu} g^{\mu'\nu'} + \\
& 4k_1 \cdot k_2 p_+ \cdot p_- g^{\mu\mu'} g^{\nu\nu'} - 4p_+ \cdot k_2 p_- \cdot k_1 g^{\mu\mu'} g^{\nu\nu'} - \\
& 4p_+ \cdot k_1 p_- \cdot k_2 g^{\mu\mu'} g^{\nu\nu'} - 4k_1 \cdot k_2 g^{\nu\nu'} p_+^{\mu'} p_-^{\mu} + \\
& 8p_- \cdot k_2 g^{\nu\nu'} p_+^{\mu'} p_-^{\mu} - 4k_1 \cdot k_2 g^{\mu'\nu'} p_+^{\nu} p_-^{\mu} + \\
& 8p_- \cdot k_2 g^{\mu'\nu'} p_+^{\nu} p_-^{\mu} + 4k_1 \cdot k_2 g^{\nu\mu'} p_+^{\nu'} p_-^{\mu} - \\
& 8p_- \cdot k_2 g^{\nu\mu'} p_+^{\nu'} p_-^{\mu} - 4k_1 \cdot k_2 g^{\nu\nu'} p_+^{\mu} p_-^{\mu'} + \\
& 4k_1 \cdot k_2 g^{\mu\nu'} p_+^{\nu} p_-^{\mu'} + 4k_1 \cdot k_2 g^{\mu\nu} p_+^{\nu'} p_-^{\mu'} - \\
& 8p_+ \cdot k_2 g^{\nu\nu'} p_-^{\mu} p_-^{\mu'} + 4k_1 \cdot k_2 g^{\mu'\nu'} p_+^{\mu} p_-^{\nu} + \\
& 4k_1 \cdot k_2 g^{\mu\nu'} p_+^{\mu'} p_-^{\nu} - 4k_1 \cdot k_2 g^{\mu\mu'} p_+^{\nu'} p_-^{\nu} +
\end{aligned}$$

$$\begin{aligned}
& 8p_+ \cdot k_2 g^{\mu'\nu'} p_-^\mu p_-^\nu + 4k_1 \cdot k_2 g^{\nu\mu'} p_+^\mu p_-^{\nu'} - \\
& 8p_- \cdot k_1 g^{\nu\mu'} p_+^\mu p_-^{\nu'} - 4k_1 \cdot k_2 g^{\mu\nu} p_+^{\mu'} p_-^{\nu'} + \\
& 8p_- \cdot k_1 g^{\mu\nu} p_+^{\mu'} p_-^{\nu'} - 4k_1 \cdot k_2 g^{\mu\mu'} p_+^\nu p_-^{\nu'} + \\
& 8p_- \cdot k_1 g^{\mu\mu'} p_+^\nu p_-^{\nu'} + 8p_+ \cdot k_1 g^{\nu\mu'} p_-^\mu p_-^{\nu'} + \\
& 8p_+ \cdot k_2 g^{\nu\mu'} p_-^\mu p_-^{\nu'} - 16p_+ \cdot p_- g^{\nu\mu'} p_-^\mu p_-^{\nu'} + \\
& 8p_+ \cdot k_1 g^{\mu\nu} p_-^{\mu'} p_-^{\nu'} + 16p_+^\nu p_-^\mu p_-^{\mu'} p_-^{\nu'} - \\
& 8p_+ \cdot k_1 g^{\mu\mu'} p_-^\nu p_-^{\nu'} + 16p_+^{\mu'} p_-^\mu p_-^\nu p_-^{\nu'} .
\end{aligned} \tag{8.28}$$

All these traces have been calculated with the use of the Mathematica program [16].

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